Technical Comments

Comment on "Dynamic Response of a Cylinder to a Side Pressure Pulse"

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THE response of a long circular elastic cylindrical shell to a cosine distributed impulsive load over half the shell circumference has recently been the subject of several analytical and experimental investigations. Traveling wave solutions for membrane stresses are presented in Refs. 1 and 2 and a modal solution for both membrane and bending stresses (based on the shell equations given in Ref. 3) is presented in Ref. 4. Unfortunately, the analysis of Ref. 4 is inaccurate in that critical terms were omitted from the frequency equation. In particular, the frequency associated with the fundamental bending mode is in considerable error. However, the solutions of Ref. 4 can easily be corrected by using the frequency equations derived in Refs. 5 and 6.

The analyses presented in Ref. 5 also suggest that a solution to the problem utilizing simplified shell equations which uncouple the extensional and inextensional vibrations should have the same range of validity as the coupled equations. Solutions for the membrane and bending stresses based on the purely extensional[1] and inextensional[7,8] equations of shell motion are

$$\frac{\sigma_m h}{Ic} = -\frac{1}{\pi}\sin\tau - \frac{1}{(2)(2)^{1/2}}\sin\left[(2)^{1/2}\tau\right]\cos\theta + \frac{2}{\pi}\sum_{n=2,4}^{\infty}\frac{(-1)^{n/2}}{(n^2-1)(n^2+1)^{1/2}}\sin\left[(n^2+1)^{1/2}\tau\right]\cos n\theta \qquad (1)$$

$$\frac{\sigma_b h}{Ic} = \frac{2(3)^{1/2}}{\pi}\sum_{n=2,4}^{\infty}\frac{n(-1)^{n/2}}{(n^2-1)(n^2+1)^{1/2}}\cdot \sin\left[\frac{n(n^2-1)}{2(3)^{1/2}(n^2+1)^{1/2}}\cdot\frac{h\tau}{a}\right]\cos n\theta \qquad (2)$$

$$\tau = ct/a, \quad c^2 = E/[\rho(1-\nu^2)] \qquad (3)$$

where σ_m is the shell membrane stress, σ_b is the bending stress at the radially outward shell surface; h and a are the shell thickness and mean radius; E, ρ, and ν are Young's modulus, density and Poisson's ratio; t is time; I is the peak intensity of the impulsive load which is cosinusoidally distributed over $|\theta| < \pi/2$; and θ is the angular coordinate.

Stress responses predicted by Ref. 4 with the corrected frequency equation were compared with the stress responses predicted by Eqs. (1) and (2) at $\theta = 0, \pi/2, \pi$ and no differences could be detected in the plotted results.‡ However, Fig. 1 demonstrates the differences between the uncorrected and corrected results of Ref. 4.

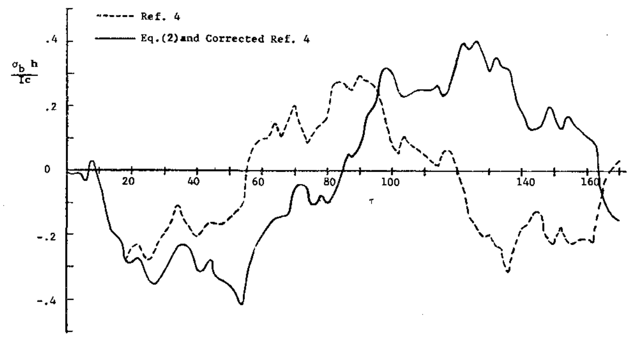

Fig. 1 Bending stress at $\theta = \pi$, $a/h = 20$.

References

[1] Payton, R. G., "Dynamic Membrane Stresses in a Circular Elastic Shell," *Journal of Applied Mechanics*, Vol. 28, No. 3, Sept. 1961, pp. 417–420.

[2] Forrestal, M. J. and Alzheimer, W. E., "Dynamic Membrane Stress in a Circular Viscoelastic Ring," *Journal of Applied Mechanics*, Vol. 36, No. 4, Dec. 1969, pp. 886–888.

[3] Flügge, W., *Stresses in Shells*, Springer-Verlag, New York, 1967, pp. 214, 219.

[4] Humphreys, J. S. and Winter, R., "Dynamic Response of a Cylinder to a Side Pressure Pulse," *AIAA Journal*, Vol. 3, No. 1, Jan. 1965, pp. 27–32.

[5] Goodier, J. N. and McIvor, I. K., "Dynamic Stability and Nonlinear Oscillations of Cylindrical Shells (Plane Strain) Subjected to Impulsive Pressure," TR 132, June 1962, Div. of Engineering Mechanics, Stanford Univ., Stanford, Calif.

[6] Goodier, J. N. and McIvor, I. K., "The Elastic Cylindrical Shell Under Nearly Uniform Radial Impulse," *Journal of Applied Mechanics*, Vol. 31, No. 2, June 1964, pp. 259–266.

[7] Timoshenko, S., *Vibration Problems in Engineering*, 3rd ed., D. Van Nostrand, Princeton, N.J., Jan. 1955, pp. 425–430.

[8] Morley, L. S. D., "Elastic Waves in a Naturally Curved Rod," *Quarterly Journal Mechanics and Applied Mathematics*, Vol. 14, Pt. 2, 1961, pp. 155–172.

‡ Plots of these curves are available from the authors.

Received December 26, 1972. This work was supported by the United States Atomic Energy Commission.
Index category: Structural Dynamic Analysis.

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FORRESTAL et al. claim that we used a modal solution, and that critical terms were omitted from the frequency equation. The modal method was not used for the solution, nor was any frequency equation presented, using the commonly accepted meaning of these terms. The solution was a series consisting of terms involving the products of a function of time and a function of the space variable, chosen to match the corresponding form of the loading function. The time-dependent portion of the solution

Received April 25, 1973. J. S. Humphreys, senior author, died in 1966.
Index category: Structural Dynamic Analysis.

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